

Modeling and Simulation of Filippov System Models with Sliding Motions using Modelica

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Abstract

This paper presents a generalized modeling formulation for implementation of dynamical system models that exhibit sliding behavior. The proposed formulation is based on Filippov theory, and is implemented using Modelica. The main advantage of the developed framework is that it effectively removes numerical chattering and trajectory deadlock. The robustness of the formulation is assessed considering three example system models: a stick-slip system, a relay feedback system and an anti-windup Proportional Integral (PI) controller in a power system application, i.e. an automatic voltage controller.

Keywords: Discontinuity, non-smooth, Filippov, hybrid dynamics, chattering.

1 Introduction

The Modelica Language is becoming an industry standard to model dynamical systems described by a set of Ordinary Differential Equations (ODEs) or Differential Algebraic Equations (DAEs) with mixed continuous and discrete variables, known as hybrid systems (Fritzson, 2014). A subclass of hybrid systems referred to as Filippov systems (Filippov, 1988) are those where discontinuities appear in vector fields, i.e. in the right hand side of the model's equations. If the solution of Filippov systems enters into a constrained subset of the state space, known as *sliding*, the formalism given by Filippov (Filippov, 1988) allows defining a vector field on the sliding surface to properly handle discontinuities.

A Modelica implementation of the Filippov systems without considering Filippov formalism leads to chattering-Zeno-type deadlocks, which consists of infinitely many instantaneous switches of the discrete variables during time domain simulation (Aljarbough and Caillaud, 2016). This significantly restrains the performance of the solvers of Modelica simulation tools and can lead to a simulation halt. More importantly, such chattering does not represent the complicated real physical system chattering (Levant, 2010). Therefore a generalized formulation is required for smooth continuation of trajectories. This paper fills this gap.

The authors in (Aljarbough and Caillaud, 2016) and (Aljarbough et al., 2016) discussed chattering problem of Fil-

ippov systems in Modelica, using OpenModelica. However, to solve this problem, a framework based on the Functional Mock-up Interface standard (FMI) and Acumen (Taha et al., 2015) is proposed. The authors in (Suski and Pytlak, 2017) presented a race car model based on the Filippov formalism implemented in OpenModelica, but did not provide implementation details and a generalized formulation. It is worth mentioning that there exist heuristic methods to solve the chattering issues (Bonilla et al., 2011). However, such methods lack a systematic generalization.

There exists several methods to formulate hybrid systems as smooth systems, for example are: linear complementarity description (Pfeiffer and Glocker, 1996); augmented Lagrangian approach (Leine and Nijmeijer, 2013); parameterized curve description (Otter et al., 1999) and then use a dedicated numerical method to solve time domain simulations. Such techniques require special treatment of numerical methods and do not model precisely discrete (non-smooth) events (Leine and Nijmeijer, 2013). Considering this, we propose a general purpose hybrid design for implementing Filippov systems using Modelica that allows smooth integration along chattering regions.

The main contributions of this paper are as follows:

- A generic formulation based on Filippov Theory (FT) for the implementation and direct numerical simulation of Filippov systems with one sliding surface using Modelica is proposed.
- A validation of the proposed formulation is performed comparing the results with a Matlab implementation and via simulation in two Modelica tools, namely OpenModelica and Dymola.

The remainder of the paper is organized as follows. Section 2 provides a background on FT. Section 3 presents the proposed generalized formulation for the implementation of Filippov system models using Modelica. Case studies are discussed in Section 4 where three examples are presented: a stick-slip system, a relay feedback system and an anti-windup PI controller in a power system voltage control application. Conclusions and future work directions are drawn in Section 5.

Dymola without numerical issues. Results are compared in Fig. 4. Observe that considering the Filippov sliding condition the simulation continues without any chattering. For further validation of the proposed generalized formulation and its implementation, the trajectories in Fig. 5 obtained using Modelica tools (OpenModelica and Dymola) and compared with the results obtained using Matlab utilizing the method in (Piiroinen and Kuznetsov, 2008).

4.3 Example 2: A relay feedback system

A relay feedback system with single-input and single-output is as follows (Piiroinen and Kuznetsov, 2008):

$$\begin{aligned} \dot{x} &= \mathbf{A}x + \mathbf{B}u \\ y &= \mathbf{C}x \\ u &= -\text{sgn}(y) \end{aligned} \quad (11)$$

or

$$\dot{x} = \begin{cases} \mathbf{A}x + \mathbf{B}, & \text{when } \mathbf{C}x < 0 \\ \mathbf{A}x - \mathbf{B}, & \text{when } \mathbf{C}x > 0 \end{cases} \quad (12)$$

where,

$$\mathbf{A} = \begin{pmatrix} -(2\zeta\omega + 1) & 1 & 0 \\ -(2\zeta\omega + \omega^2) & 0 & 1 \\ -\omega^2 & 0 & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ -2\sigma \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

The state vector of this model is $x = [x_1, x_2, x_3]^T$ and the discontinuity surface Σ is defined by $h(x) = x_1$. Re-writing the dynamical system according to FT,

$$\dot{x} = f(x) = \begin{cases} f_1(x) & \text{when } h(x) < 0 \\ f_2(x) & \text{when } h(x) > 0 \end{cases} \quad (13)$$

with

$$f_1(x) = \begin{pmatrix} -(2\zeta\omega + 1)x_1 + x_2 + 1 \\ -(2\zeta\omega + \omega^2)x_1 + x_3 - 2\sigma \\ -\omega^2x_1 + 1 \end{pmatrix},$$

$$f_2(x) = \begin{pmatrix} -(2\zeta\omega + 1)x_1 + x_2 - 1 \\ -(2\zeta\omega + \omega^2)x_1 + x_3 + 2\sigma \\ -\omega^2x_1 - 1 \end{pmatrix}.$$

Here, $h_x(x) = [1 \ 0 \ 0]^T$, thus on Σ (i.e. $x_1 = 0$), calculating,

$$\begin{aligned} r_1 &= -(2\zeta\omega + 1)x_1 + x_2 + 1, \\ r_2 &= -(2\zeta\omega + 1)x_1 + x_2 - 1. \end{aligned}$$

The sliding vector field on Σ obtained using equations (8,9):

$$\alpha(x) = (-(2\zeta\omega + 1)x_1 + x_2 + 1)/2,$$

$$f_F(x) = \begin{pmatrix} 0 \\ b + 4(-(2\zeta\omega + 1)x_1 + x_2 + 1)/2 \\ c - 2(-(2\zeta\omega + 1)x_1 + x_2 + 1)/2 \end{pmatrix},$$

where $b = -(2\zeta\omega + \omega^2)x_1 + x_3 - 2\sigma$ and $c = -\omega^2x_1 + 1$.

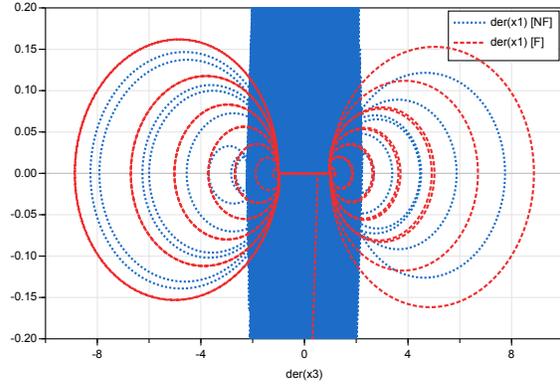


Figure 6. Time derivative of state variable (\dot{x}_1) of the relay feedback system model without (NF) and with (F) Filippov theory simulated in Dymola.

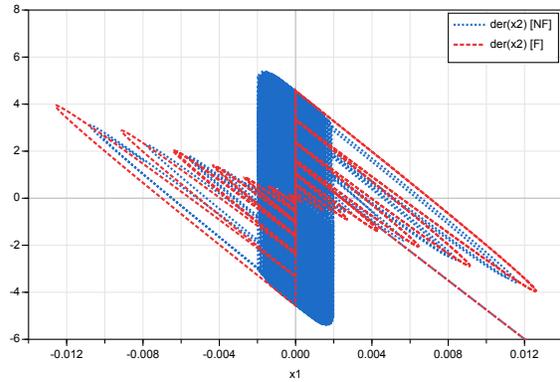


Figure 7. Time derivative of state variable (\dot{x}_2) of the relay feedback system model without (NF) and with (F) Filippov theory simulated in Dymola.

Similarly to the previous example, we have implemented both in direct form and using the proposed formulation. The parameters considered are: $\zeta = 0.05$, $\omega = 25$ and $\sigma = -1$. Using the Modelica language similar issues (chattering and trajectory deadlock) as in the previous example arise in OpenModelica and Dymola during time domain simulation without considering FT and the simulation results obtained using Dymola are shown in Figs. 6-7. Observe that chattering does not occur for the model implemented following the proposed approach based on FT. Finally the validation of the results of Modelica tools against the implementation in Matlab (Piiroinen and Kuznetsov, 2008) is shown in Fig. 8.

4.4 Example 3: Anti-windup PI controller

The IEEE Standard 421.5-2016 recommends an anti-windup (AW) or non-windup PI controller (IEEE, 2016) model for dynamic analysis of power systems. Mathematically, the model is:

$$\begin{aligned} \text{If } y &\geq w^{\max} : w = w^{\max} \text{ and } \dot{x} = 0, \\ \text{If } y &\leq w^{\min} : w = w^{\min} \text{ and } \dot{x} = 0, \\ \text{Otherwise} & : w = y = k_p u + x \text{ and } \dot{x} = k_i u. \end{aligned} \quad (14)$$

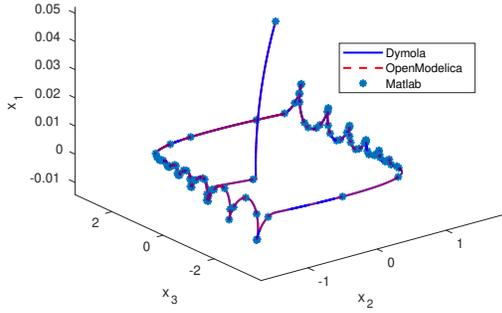


Figure 8. State space response for the relay feedback system obtained in different simulation software tools.

where k_p , k_i , are the proportional, integral gains of the controller, respectively; x is the controller's state variable; and u , y and w are the control input, unconstrained output and constrained output respectively.

The discontinuous nature of the integral state variable of this model can lead to numerical issues such as trajectory deadlock and chattering. Only *ad hoc* approaches have been proposed to handle such issues (Hiskens, 2012; Fabozzi et al., 2017). In the authors preliminary results, it was shown that the formalism given by Filippov can effectively remove those numerical issues (Murad et al., 2019). In this work, the AW PI controller is utilized in a power system voltage control application namely an Automatic Voltage Controller (AVR), which is implemented using the proposed formulation in Modelica. In addition, the dynamic response is compared to a deadband (DB) based technique proposed in (Hiskens, 2012) and it is shown that the proposed approach achieves a smooth transient response.

4.4.1 Single machine infinite bus system

Consider a simple three bus power system network with a single machine shown in Fig. 9. The generator in bus 1 is equipped with an AW PI controller based AVR and a power system stabilizer as depicted in Fig. 10. The dynamics of this Single Machine Infinite Bus (SMIB) system is described by a set of DAEs in the following form (Milano, 2010),

$$\begin{aligned} \dot{x} &= f(x, y), \\ 0 &= g(x, y), \end{aligned} \quad (15)$$

where x and y are the vector of state and algebraic variables respectively.

Then generator in Fig. 9 (Gen) is modelled using a third order model (Milano, 2010), and the switching manifold for the maximum of the AVR, $h(x) = k_p v_a + x_i - v^{\max}$. When $h(x) < 0$, the differential equations of the SMIB system are:

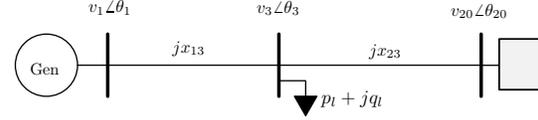


Figure 9. A single generator connected to an infinite bus.

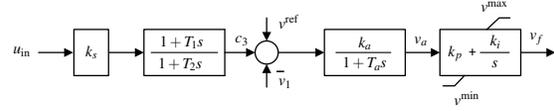


Figure 10. Scheme of AVR and power system stabilizer.

$$\dot{\delta} = \omega \quad (16)$$

$$\dot{\omega} = \frac{1}{M}(p_m - p_e - D\omega) \quad (17)$$

$$\dot{e}'_q = \frac{1}{T'_{d0}}(v_f - \frac{x_d}{x'_d} e'_q + \frac{x_d - x'_d}{x'_d} v_1 \cos(\delta - \theta_1)) \quad (18)$$

$$\dot{v}_a = (k_a(v^{\text{ref}} + c_3 - v_1) - v_a)/T_a \quad (19)$$

$$\dot{x}_i = k_i v_a \quad (20)$$

$$\dot{s}_1 = \frac{1}{T_2}(c_2 - s_1). \quad (21)$$

The notations and algebraic equations are given in (Murad et al., 2019). When the v_f reaches to its max (v^{\max}) then (18) and (20) will be switched and all other state variables will remain same, as follows:

$$\dot{e}'_q = \frac{1}{T'_{d0}}(v^{\max} - \frac{x_d}{x'_d} e'_q + \frac{x_d - x'_d}{x'_d} v_1 \cos(\delta - \theta_1)) \quad (22)$$

$$\dot{x}_i = 0 \quad (23)$$

4.4.2 Implementation using Filippov theory

We consider $f_1(x, y)$ is (16)-(21) and $f_2(x, y)$ is (16), (17), (22), (19), (23) and (21). Calculating, $h_x(x) = [\frac{\partial h(x)}{\partial x_1} \quad \frac{\partial h(x)}{\partial x_2} \quad \dots \quad \frac{\partial h(x)}{\partial x_6}]^T = [0 \ 0 \ 0 \ k_p \ 1 \ 0]^T$, and the normal to the switching surface is: $n^T(x) = [0 \ 0 \ 0 \ k_p \ 1 \ 0]$. On the switching manifold, calculating

$$h_x^T(x) f_1(x, y) = k_p((k_a(v^{\text{ref}} + c_3 - v_1) - v_a)/T_a) + k_i v_a,$$

$$h_x^T(x) f_2(x, y) = k_p((k_a(v^{\text{ref}} + c_3 - v_1) - v_a)/T_a).$$

Therefore according to (6), if an attractive sliding occurs on Σ , then using equation (8) $\alpha(x, y)$ is given by:

$$\alpha(x, y) = \frac{k_p((k_a(v^{\text{ref}} + c_3 - v_1) - v_a)/T_a) + k_i v_a}{k_i v_a}.$$

According to (9), during the sliding requires (23):

$$f_F(x, y) = -k_p((k_a(v^{\text{ref}} + c_3 - v_1) - v_a)/T_a).$$

These expressions are used in the Modelica implementation.

Table 1. PARAMETERS OF THE SMIB NETWORK

Name	Values
Generator	$M = 8, D = 0, x'_d = 0.25, x_d = 1,$ $p_m = 1, T'_{d0} = 6$
Line	$x_{13} = 0.3, x_{23} = 0.5$
Load	$p_{l0} = 0.7, q_{l0} = 0.01$
AVR	$k_a = 2, T_a = 0.005, k_p = 5.5, k_i = 35,$ $v^{\max} = 1.6, v^{\min} = -1.5, v^{\text{ref}} = 1$
PSS	$k_s = 1.5, T_1 = 0.23, T_2 = 0.12$

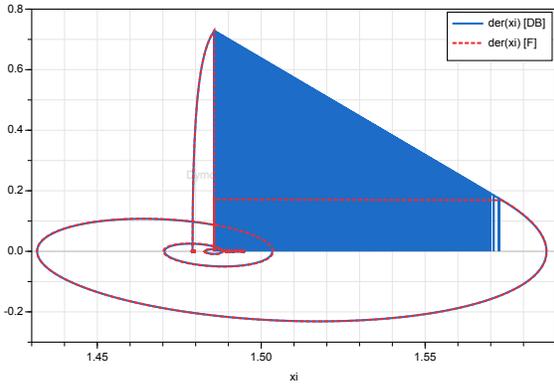


Figure 11. Time derivative of the integrator state variable (\dot{x}_i) in the AW PI controller with respect to the state variable (x_i) using DB and Filippov (F) methods simulated in Dymola.

4.4.3 Simulation Results

The SMIB system is implemented considering the FT-based formulation and the DB based method in Modelica. The DB implementation is the same as in (Hiskens, 2012) and DB value used is 0.0001. The parameters of different components of the SMIB system are given in Table 1 and initial values for all variables are given in (Murad et al., 2019).

The SMIB system is simulated by increasing the voltage reference set-point ($v^{\text{ref}} = 1.01$) and load ($p_{l0} = 0.71$ pu, $q_{l0} = 0.016$ pu) at $t = 5$ s. Figs. 11 and 12 show the response of the time derivative of integrator state variable (\dot{x}_i) and field voltage (v_f) for both DB and FT based method's respectively. Following the disturbance, the integrator state variable (x_i) enters into a deadlock region and using the DB based implementation it shows chattering. Therefore the field voltage (v_f) shows numerous switching bounded by the DB (see zoom in Fig. 12). However, a smooth response is achieved using FT based method. In addition, except for the chattering the FT based model shows the same trajectories. It is important to mention that without DB or FT based techniques simulation in OpenModelica fails for all solvers while DASSL fails for Dymola.

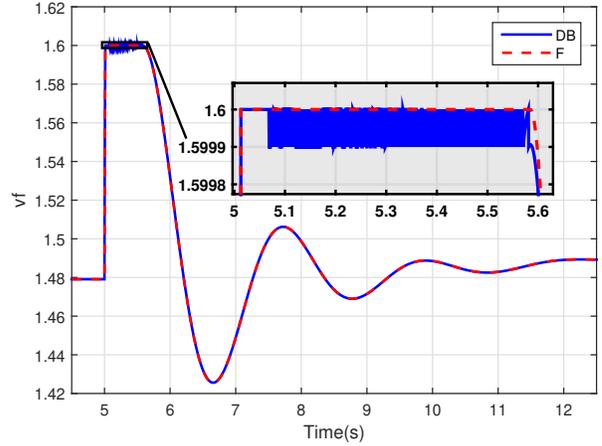


Figure 12. Trajectories of the field voltage (v_f) using DB and Filippov (F) methods simulated in Dymola.

5 Conclusions

A generic formulation to implement Filippov system models with sliding motion using Modelica is proposed. Three examples are presented considering such a general-purpose design with a single sliding surface. Simulation results in different Modelica tools indicate accurate dynamic response without any chattering or simulation halt.

Future work will extend the FT-based design for multiple discontinuity surface (Piiroinen and Kuznetsov, 2008). In addition a numerical performance of FT-based AW PI controller will be investigated considering the Modelica power system library: OpenIPSL (Vanfretti et al., 2016).

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