FAST SIMULATIONS OF AIR CONDITIONING SYSTEMS USING SPLINE-BASED TABLE LOOK-UP METHOD (SBTL) WITH ANALYTIC JACOBIANS

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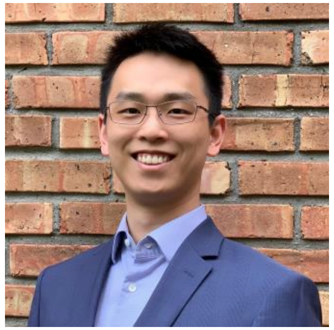
For The American Modelica Conference 2020

THE "FAST AC" TEAM



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OUTLINE

- Introduction
- Analytic Jacobians in System Simulation
- Spline-Based Table Look-Up Method (SBTL) Overview
- Implementation of Refrigerant Property Model with Analytic Jacobians
- Speedup in System Simulations

INTRODUCTION

- Property calculation is at the core of speeding up simulation of two-phase systems
  - Multi-parameter equation of state (EOS) involves costly function evaluation
  - Iteration is needed if EOS and the system model use different sets of state variables
  - Significant numbers of function evaluations
- Applied Spline-Based Table Look-Up (SBTL) Method (Kunick et al, 2015) to refrigerant property models in Modelica (Li et al., 2018) → ***2x speedup in complex system models***

INTRODUCTION

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- Applied Spline-Based Table Look-Up (SBTL) Method (Kunick et al, 2015) to refrigerant property models in Modelica (Li et al., 2018) → ***2x speedup in complex system models***
- Up to 100x speedup in very complex thermal power models with analytic Jacobians in Modelon's Thermal Power Library
- Easier to write derivative functions for quadratic splines than for Helmholtz energy EOS

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ANALYTIC JACOBIANS IN SYSTEM SIMULATION

- The Jacobian matrix is the matrix of all first-order partial derivatives of a vector valued function $\boldsymbol{f}(\boldsymbol{x})$ with respect to all inputs

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

- Fundamental for solving equations for dynamic systems
  - Nonlinear equation solved by Newton-Raphson method

$$\boldsymbol{x}_{n+1} = \boldsymbol{x}_n - \boldsymbol{J}(\boldsymbol{x}_n)^{-1}\,\boldsymbol{f}(\boldsymbol{x}_n)$$

  - Numerical integration

    e.g. DASSL solver (time derivative approximated by BDF and the equation is solved by a modified Newton's method)

ANALYTIC JACOBIANS IN SYSTEM SIMULATION

- Analytic vs. numerical Jacobians
  - More accurate derivatives, suppressing numerical noises
  - Faster convergence
  - Potentially more robust models

- Modelica compilers apply automatic differentiation (AD) to construct analytic Jacobians

- Numerical Jacobians are calculated if AD fails
  - Multi-line Modelica functions without appropriate annotations
  - External functions without derivatives

➡ ***Add derivatives to property functions!!!***

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SBTL OVERVIEW – THE SPLINE

- A specific type of quadratic/biquadratic spline on an equidistant grid (Späth, 1995)
- Advantages
  - Equidistant grid — ***Searching***
  - Continuous first derivatives
  - Analytic inverse — ***Iterations***
  - Consistent phase boundary definition — ***Chattering***

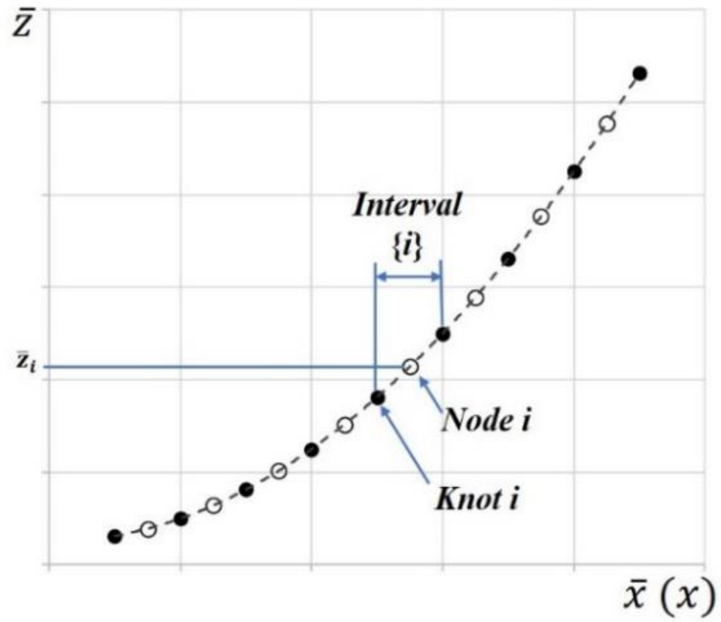

Illustration of the spline interpolation. Knots are represented by solid dots and node by hollow ones.

SBTL OVERVIEW – REFRIGERANT PROPERTY MODEL

- Phase boundary described by 1D spline: $T_s(p)$
- Single-phase region:
  - T, ρ, s given by 2D splines: $T(p,h), \rho(p,h), s(p,h)$
  - $h_v{}^{INV}(p, T_s(p))$ and $h_l{}^{INV}(p, T_s(p))$ by 2D inverse functions from $T(p,h)$ spline for consistency

    → **Avoid chattering across phase boundary**
- Two-phase region: Mass specific properties calculated based on vapor quality $x = \frac{(h - h_l)}{(h_v - h_l)}$
- Other properties like specific heat capacity can derived from the 1D and 2D splines (Tummescheit, 2002; Thorade & Saadat, 2013)

SBTL OVERVIEW – DERIVATIVE OF THE SPLINES WITH GRID TRANSFORMATION

- Highly non-linear functions → Substantial increase in number of nodes
- End up in larger data files, especially for 2D splines, data file size ~$O({n_{node}}^2)$
- Use proper coordinate transformation, e.g. $\bar{p} = \log(p)$

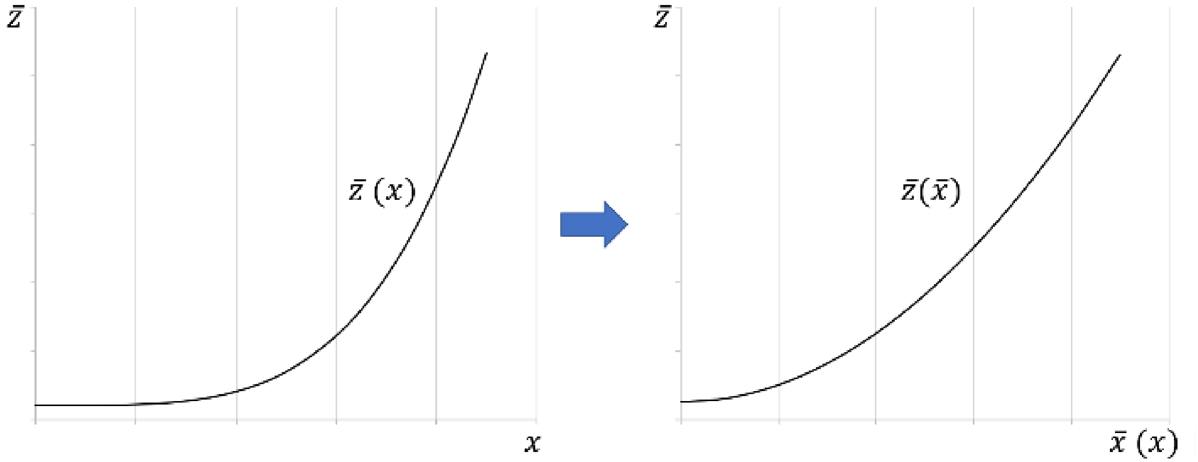

Illustration of coordinate transformation to enhance accuracy with equidistant nodes.

SBTL OVERVIEW – DERIVATIVE OF THE SPLINES WITH GRID TRANSFORMATION

- Use chain rule for derivatives

$\bar{p} = \log(p)$

$T_{s\{i\}} = a_{i1} + a_{i2}(\bar{p} - \bar{p}_i) + a_{i3}(\bar{p} - \bar{p}_i)^2$

$\bar{p}_i$ is the i^{th} transformed pressure node
a_{ik} are the spline coefficients in the i^{th} interval of the spline

1st order derivative in the i^{th} interval:

$$\frac{dT_{s\{i\}}}{dp} = \frac{dT_{s\{i\}}}{d\bar{p}}\frac{d\bar{p}}{dp} = \frac{dT_{s\{i\}}}{d\bar{p}}\frac{1}{p}$$

where

$$\frac{dT_{s\{i\}}}{d\bar{p}} = a_{i2} + 2a_{i3}(\bar{p} - \bar{p}_i)$$

2nd order derivative in the i^{th} interval:

$$\frac{d^2T_{s\{i\}}}{dp^2} = \frac{d}{dp}\left(\frac{dT_{s\{i\}}}{d\bar{p}}\frac{1}{p}\right) = \left(\frac{d^2T_{s\{i\}}}{d\bar{p}^2} - \frac{dT_{s\{i\}}}{d\bar{p}}\right)\frac{1}{p^2}$$

where

$$\frac{dT_{s\{i\}}}{d\bar{p}} = a_{i2} + 2a_{i3}(\bar{p} - \bar{p}_i)\,, \quad \frac{d^2T_{s\{i\}}}{d\bar{p}^2} = 2a_{i3}$$

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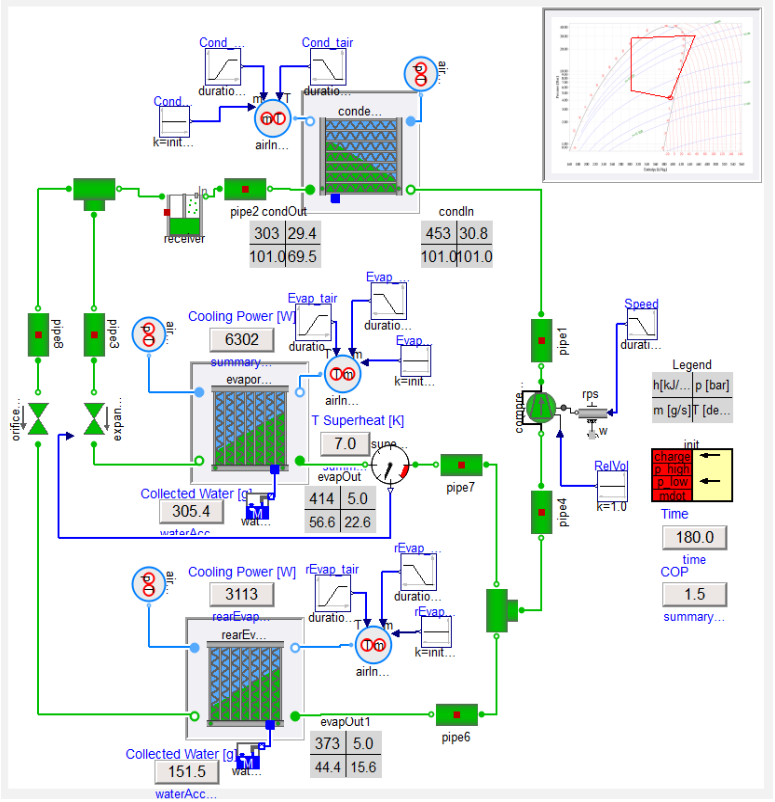
IMPLEMENTATION – DIAGNOSTICS OF REQUIRED DERIVATIVE FUNCTIONS

- Turn on two advanced flags in Dymola
  - *Advanced.GenerateAnalyticJacobian = true*
  - *Advanced.PrintFailureToDifferentiate = true*

- Compile a typical system model and investigate the translation log

- Identify missing derivatives, in our case:
  - Partial derivatives of density $\left.\frac{\partial\rho}{\partial h}\right|_p, \left.\frac{\partial\rho}{\partial p}\right|_h$
  - Saturation properties: h_l, h_v, ρ_l, ρ_v
  - Isentropic enthalpy $\mathrm{h}_{isen}(p, s, X)$
  - Functions for compressor efficiencies



IMPLEMENTATION – DERIVATIVES OF SATURATION PROPERTIES

- ${h_v}^{INV}(p, T_s(p))$ and ${h_l}^{INV}(p, T_s(p))$ as inverse functions of the 2D $T(p,h)$ spline

  (h_v and h_l are not implemented as 1D splines of pressure!!!)

  → Their derivatives must be constructed from derivatives of the $T(p,h)$ and $T_s(p)$

$$\frac{\mathrm{d}h}{\mathrm{d}p} = \left.\frac{\partial h}{\partial p}\right|_T + \left.\frac{\partial h}{\partial T}\right|_p \frac{\mathrm{d}T_s}{\mathrm{d}p}$$

where

$$\left.\frac{\partial h}{\partial p}\right|_T = -\frac{\partial T/\partial p|_h}{\partial T/\partial h|_p}, \quad \left.\frac{\partial h}{\partial T}\right|_p = \frac{1}{\partial T/\partial h|_p}$$

- Second-order derivatives of saturation properties are also needed when calculating the derivatives of $\left.\frac{\partial \rho}{\partial h}\right|_p$ and $\left.\frac{\partial \rho}{\partial p}\right|_h$

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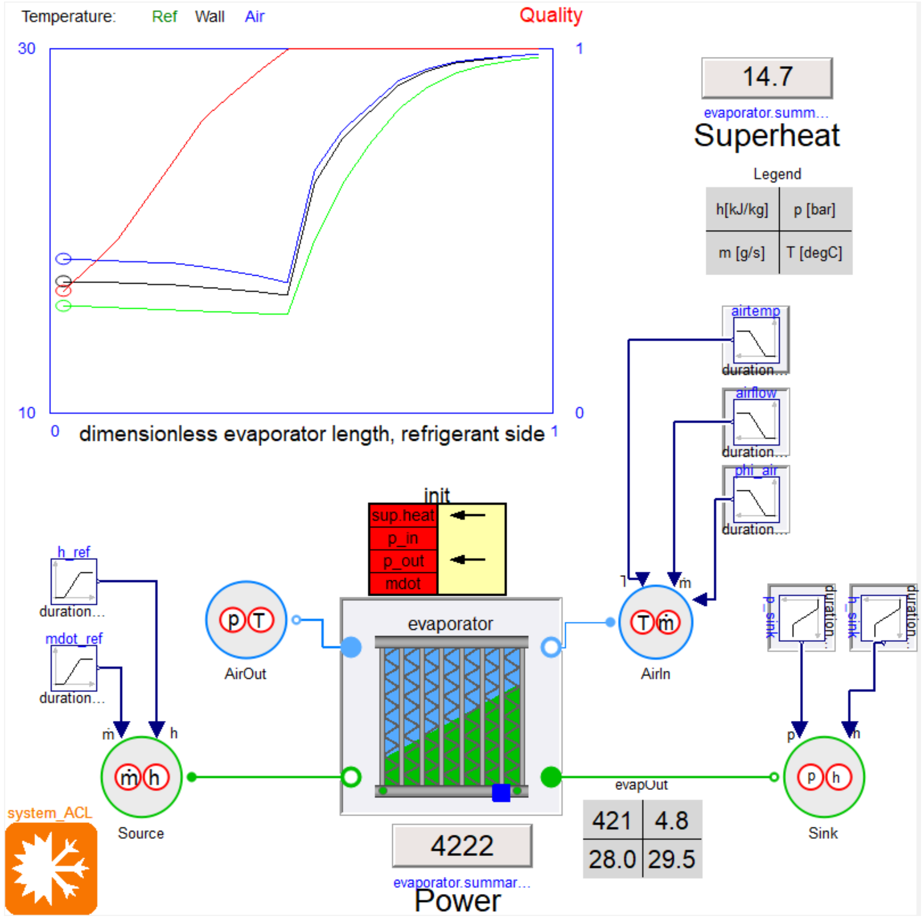
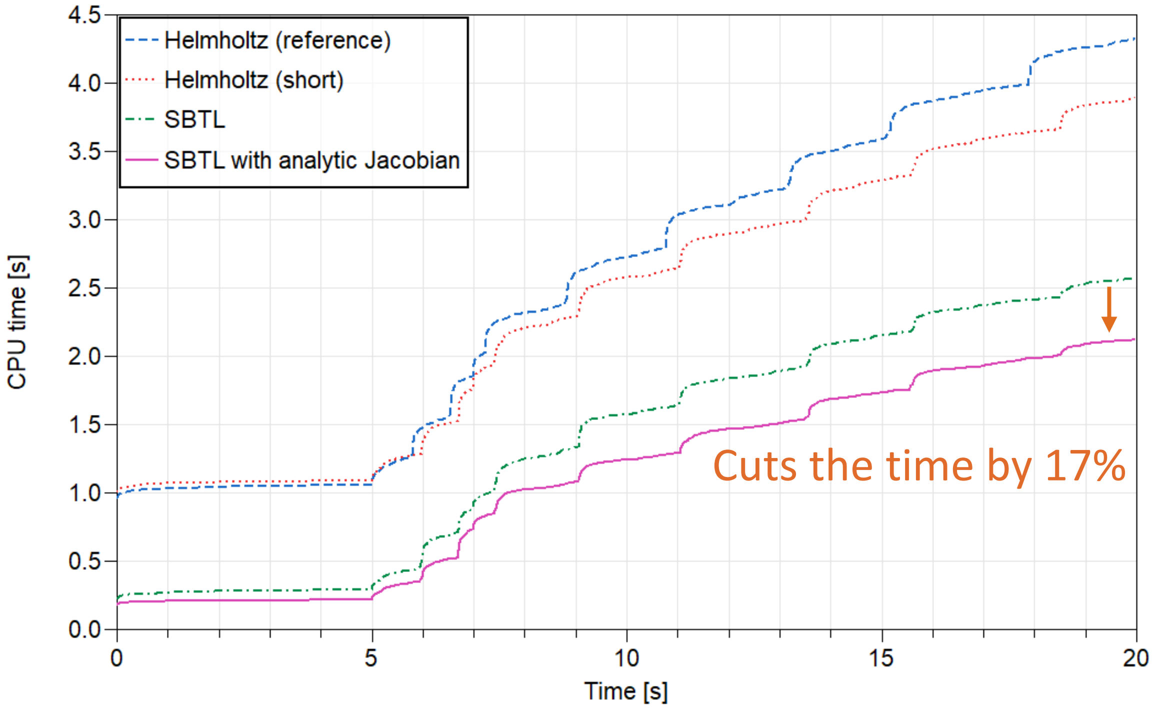
SYSTEM SIMULATIONS – HARDWARE & SOFTWARE CONFIGURATION

Model	Dell XPS 8700 Desktop
Processor	Intel® Core™ i7-4770 CPU
RAM	16.0 GB
System	64-bit, x64 based, Windows 10 Pro
Software	Dymola 2018 FD01
C compiler	Visual Studio 2015 Express Edition
Solver	Dassl
Tolerance	1e-6 (to ensure mass conservation)

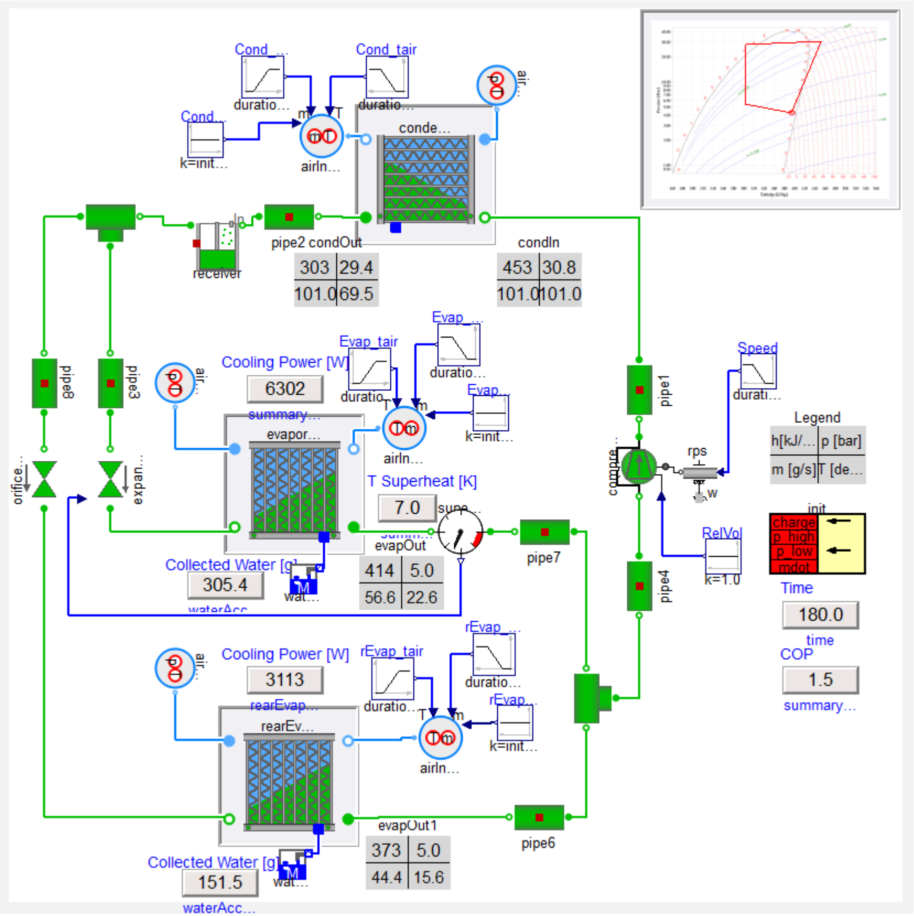
SYSTEM SIMULATIONS – EVAPORATOR TEST

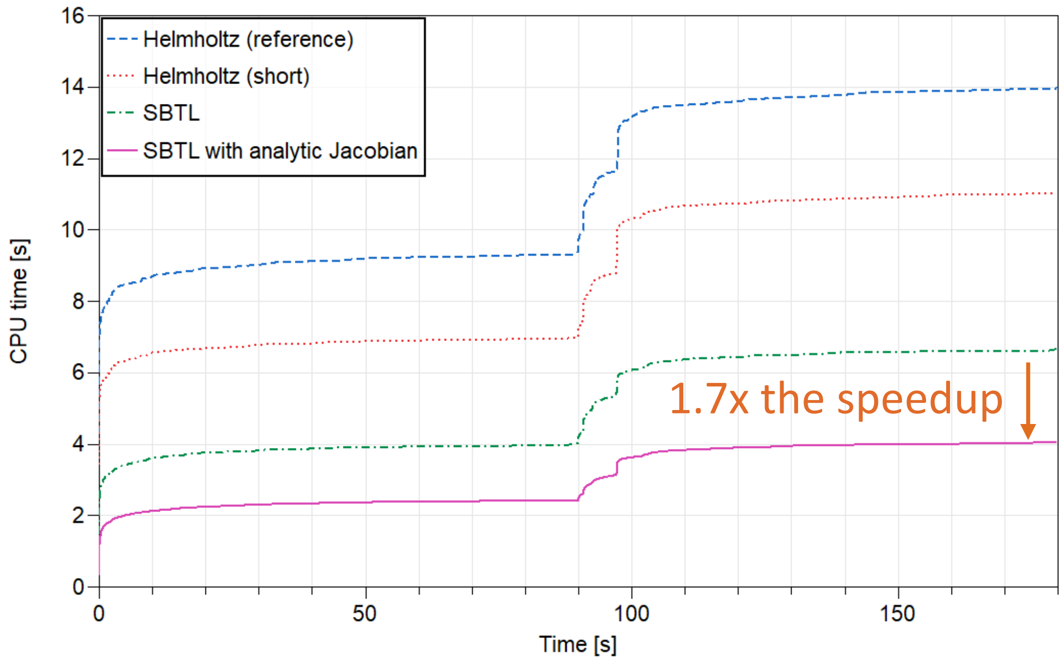




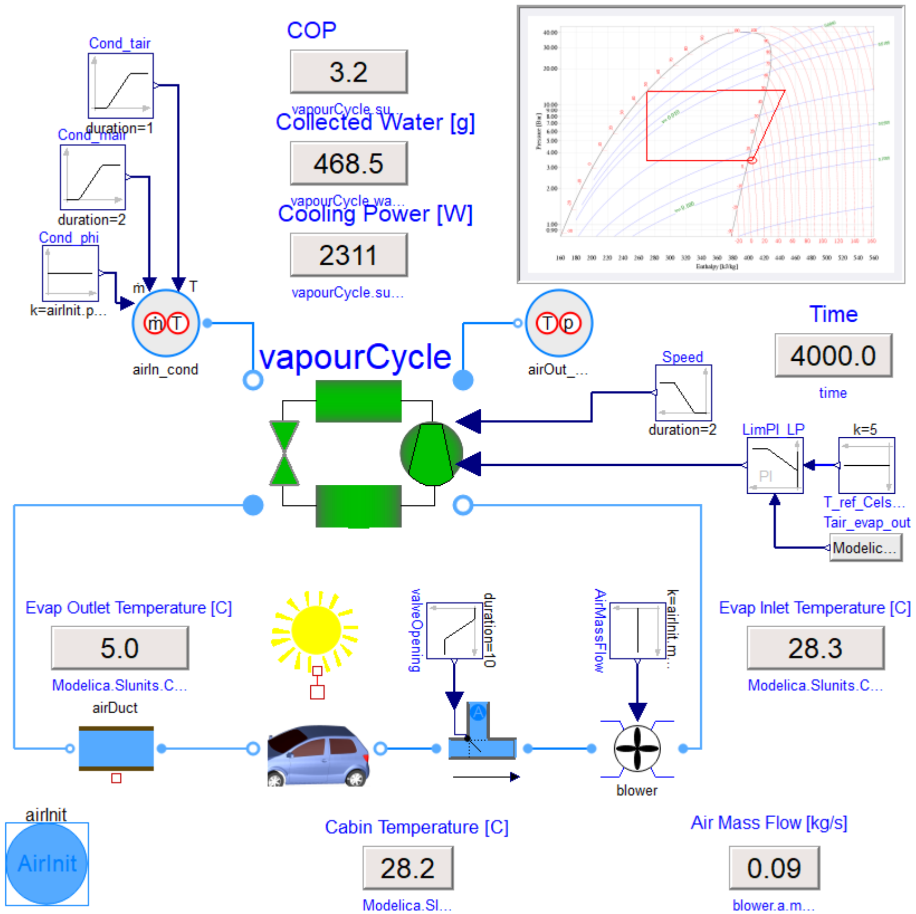
Two-layer evaporator with 18 discretized volumes & 56 dynamic states

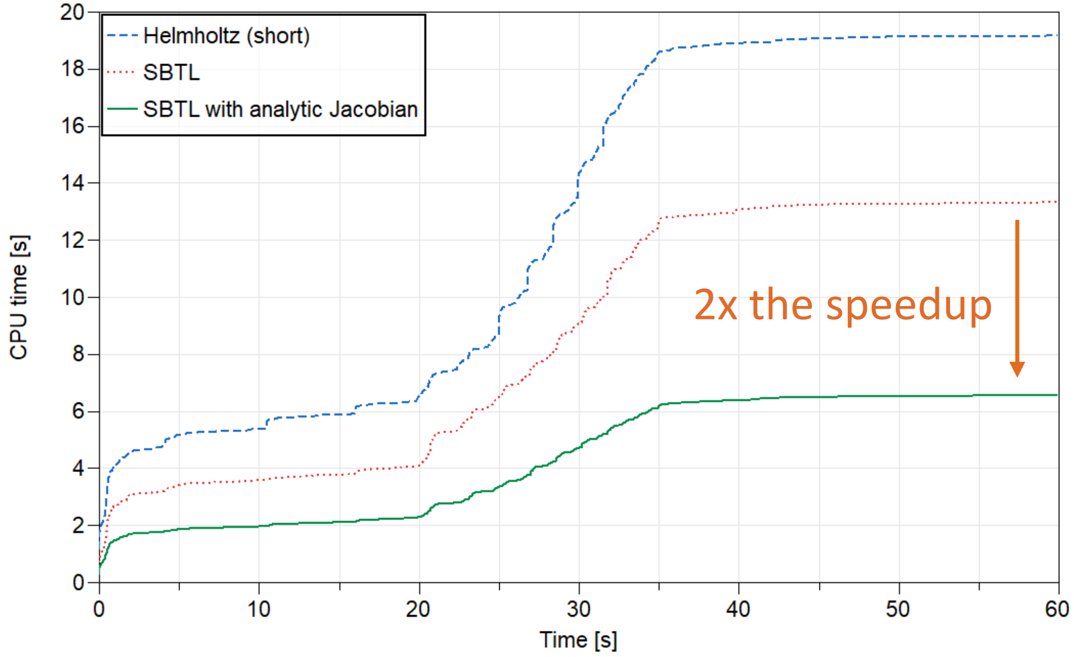
SYSTEM SIMULATIONS – TWIN EVAPORATOR CYCLE FROM ACL



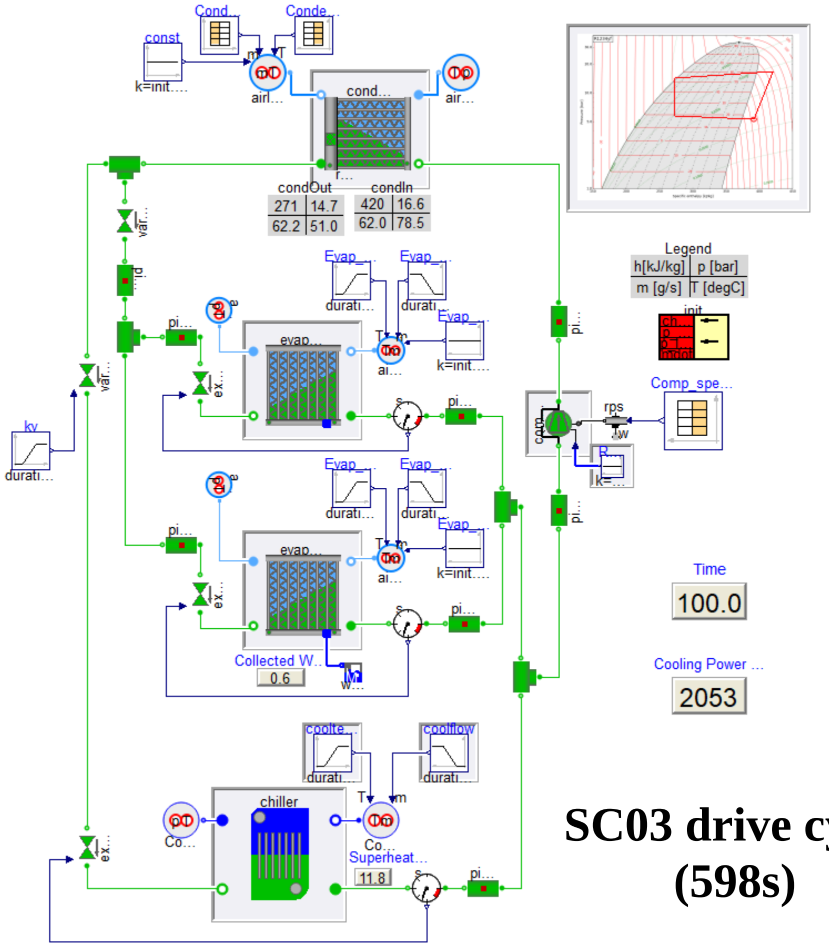


SYSTEM SIMULATIONS – AC PULL-DOWN TEST FROM ACL

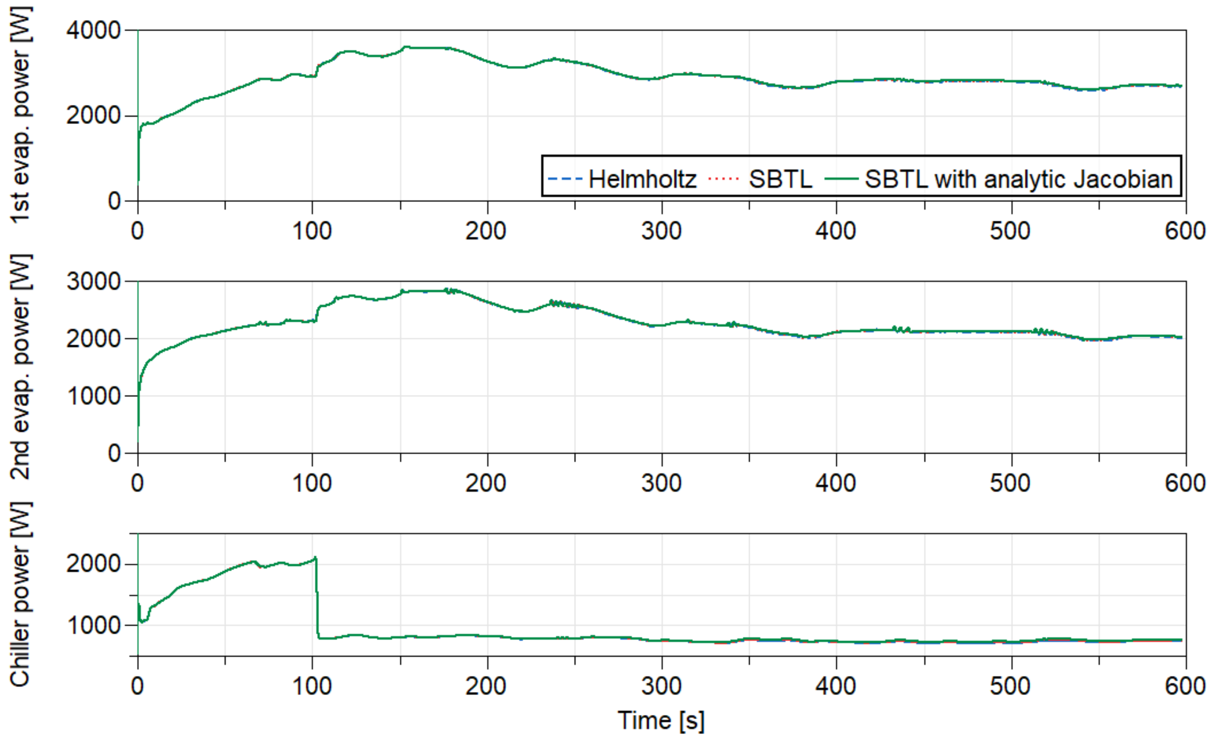




SYSTEM SIMULATIONS – R1234YF CYCLE FROM FORD MOTOR COMPANY



SC03 drive cycle (598s)



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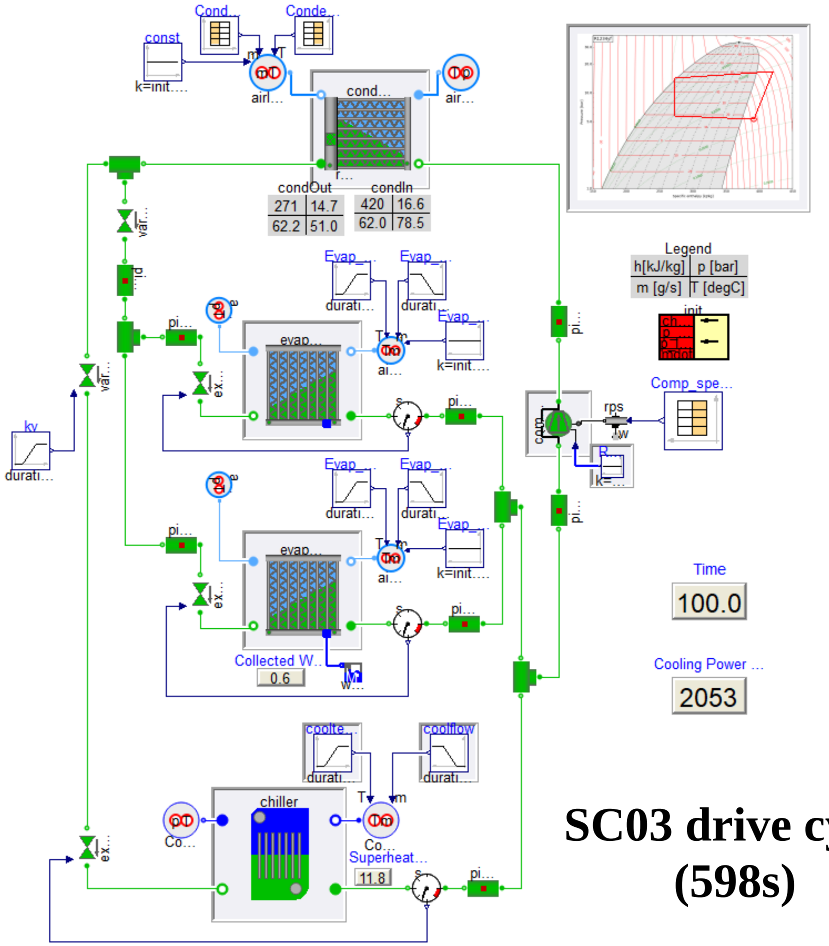

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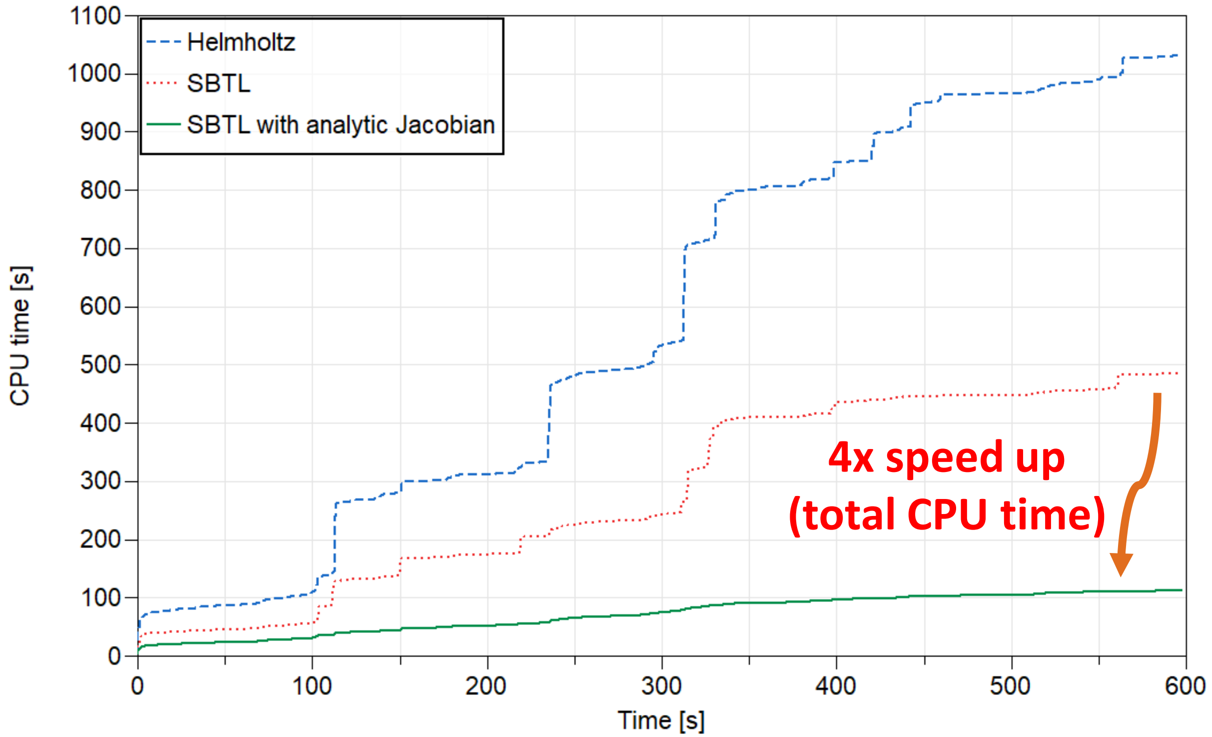

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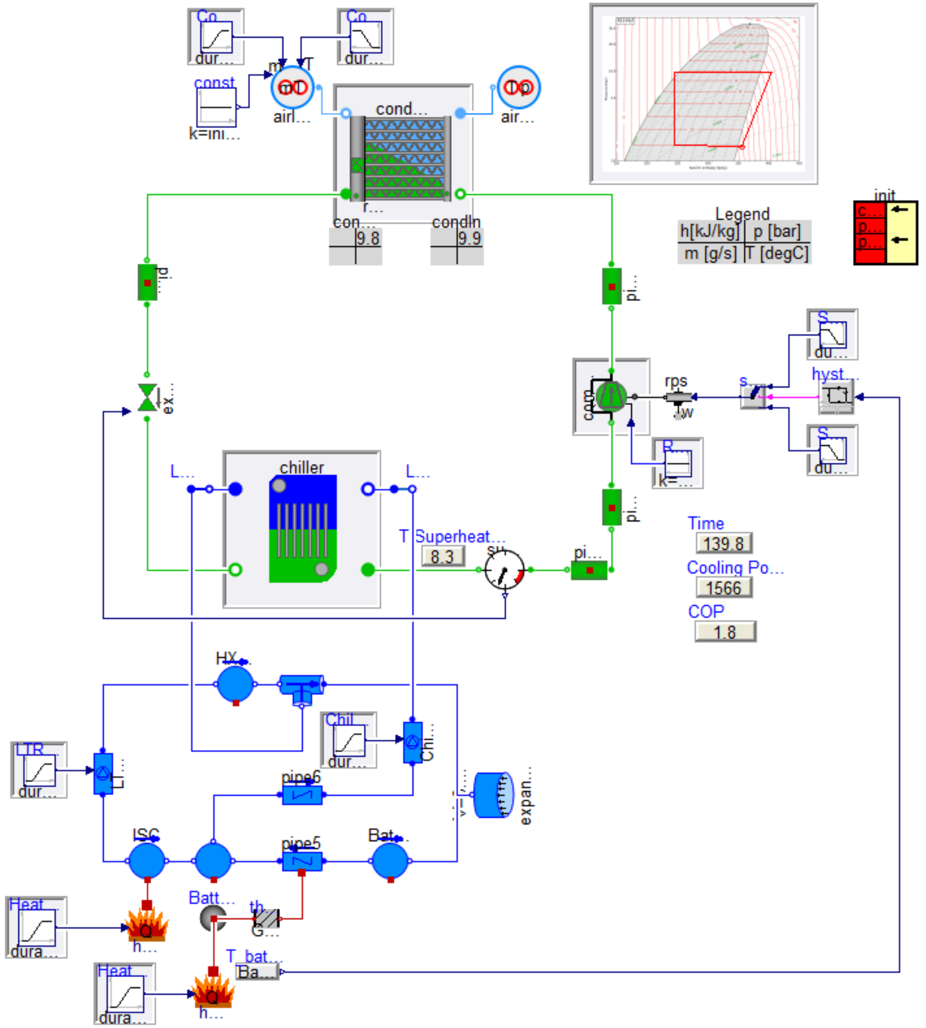

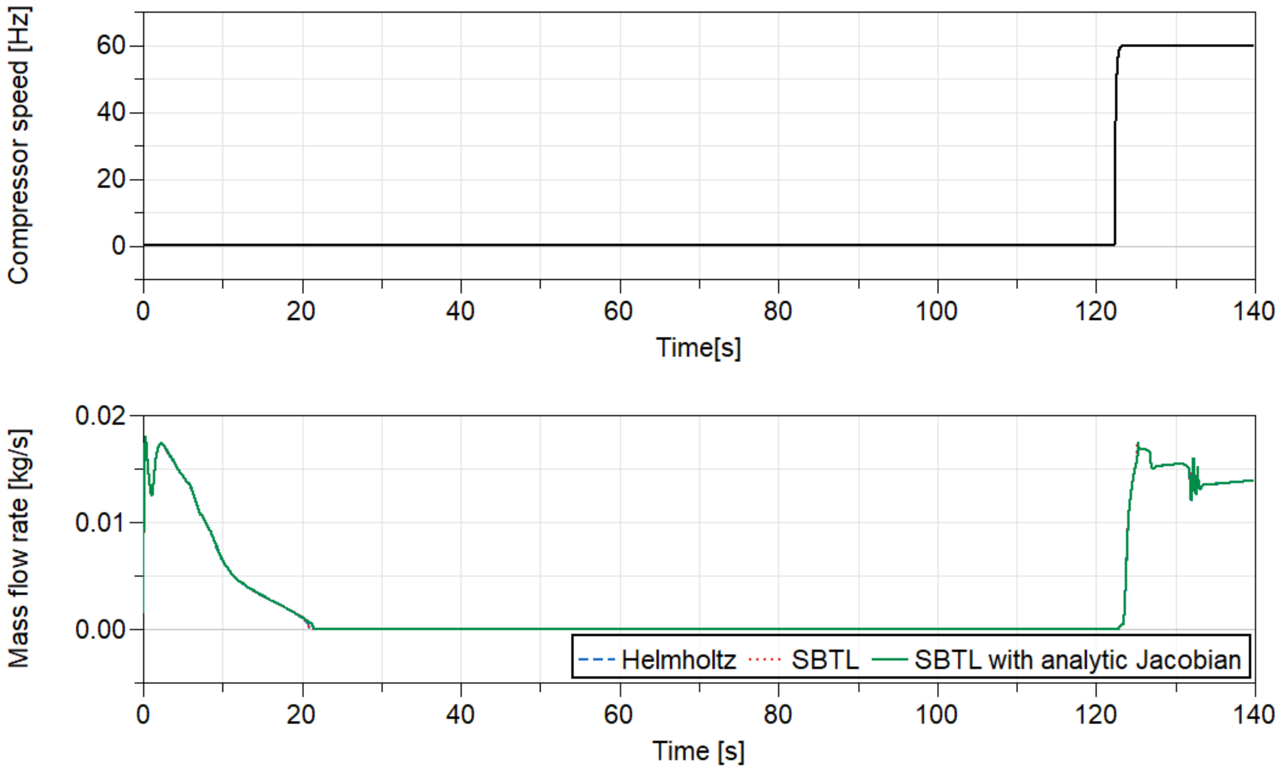

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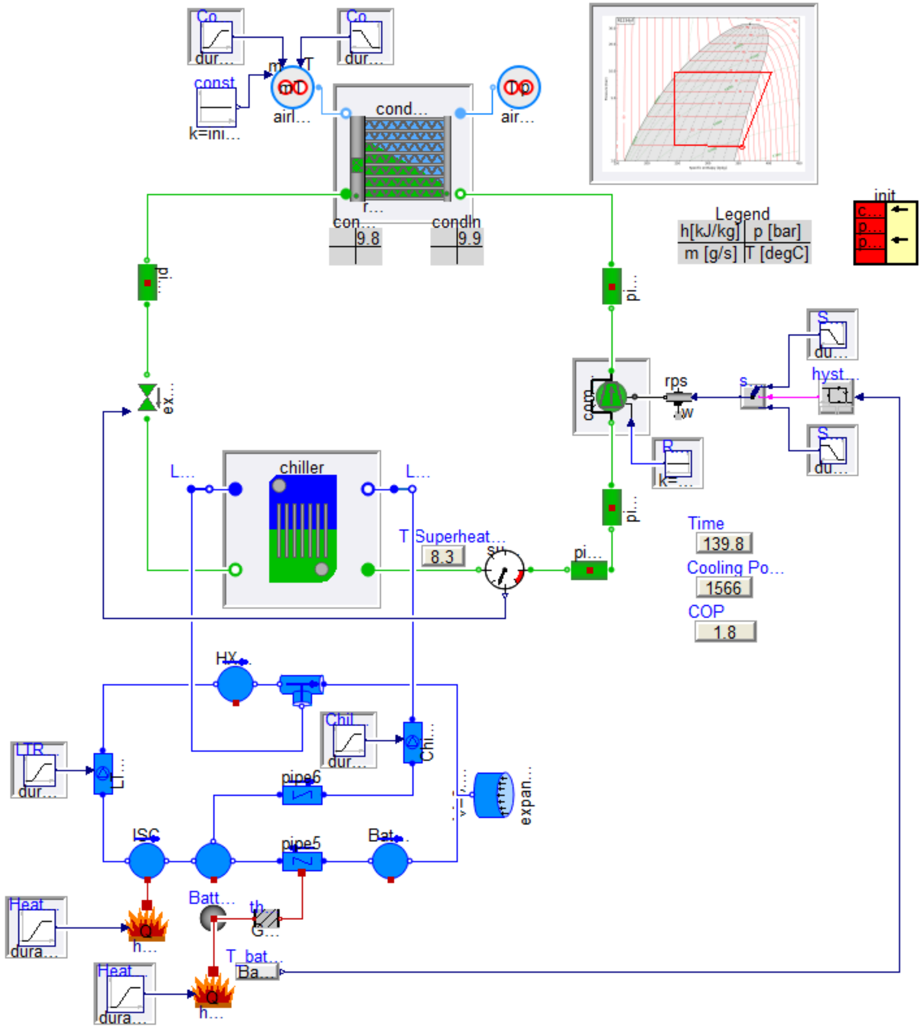

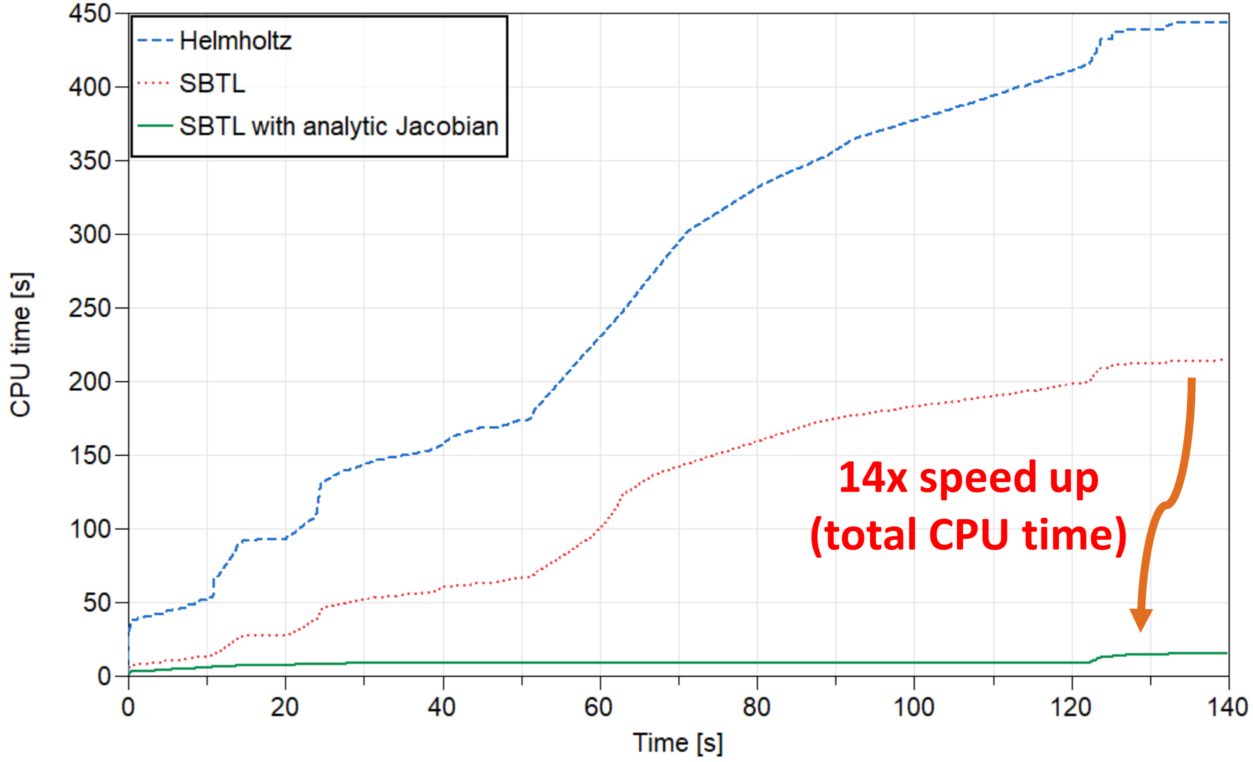

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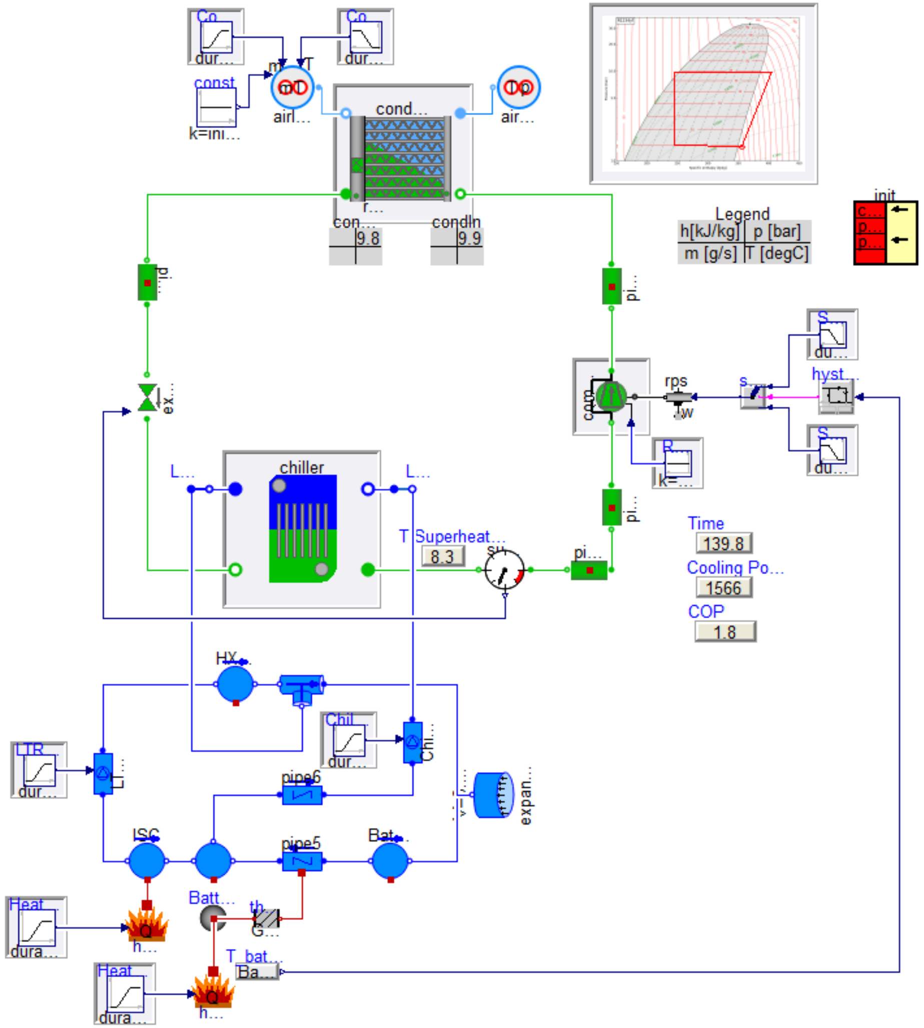

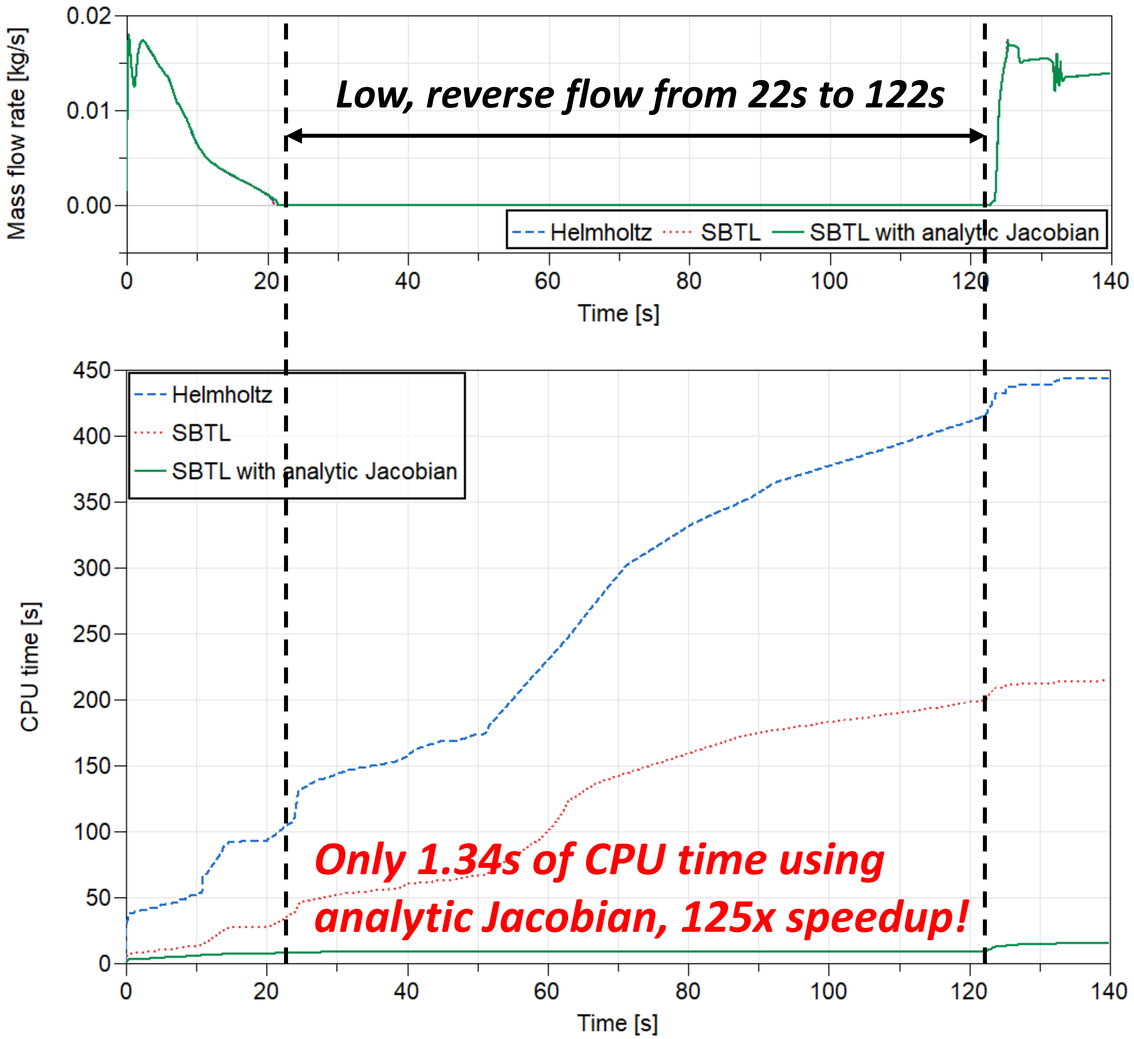

CONCLUSIONS

- Further development of the SBTL method for fast calculation of refrigerant properties in Modelica to allow the generation of analytic Jacobians.
- Reductions in computational speed ranging from to 2-4x are obtained on complex system models for SBTL with analytic Jacobians versus SBTL without analytic Jacobians.
- Even greater improvement was observed in models under low or reverse flow conditions. In addition, the computational improvement could be even larger for more complex models with even more states.

The SBTL models with analytic Jacobians for R134a and R1234yf will be available in the upcoming release of Modelon's Air Conditioning Library!

REFERENCES

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