Modeling and Simulation of Filippov System Models with Sliding Motions using Modelica

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Outline

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Motivation

- Modelica is an industry standard to model hybrid (discrete-continuous) dynamical systems described by ODEs or DAEs.
- A subclass of hybrid systems referred to as **Filippov systems** where discontinuities appear in the right hand side of the model's equations.
- A Modelica implementation of the Filippov systems without considering Filippov formalism leads to chattering-Zeno-type deadlocks, which consists of infinitely many instantaneous switches of the discrete variables during time domain simulation.
- This significantly restrains the performance of the solvers of Modelica simulation tools and can lead to a simulation halt.
- A generalized formulation is required for smooth continuation of trajectories of Fillippov system models in Modelica tools.

Objectives

- To propose a generic formulation based on Filippov Theory (FT) for the implementation and direct numerical simulation of Filippov systems with one sliding surface using Modelica.
- To validate the proposed formulation comparing the results with a Matlab implementation and via simulation in two Modelica tools, namely OpenModelica and Dymola.

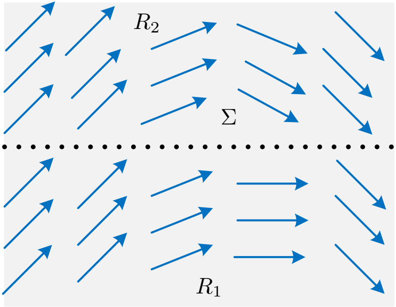
Filippov Theory

- Consider the following switched dynamical system:

$$\dot{x} = f(x) = \begin{cases} f_1(x) & \text{when } h(x) < 0 \\ f_2(x) & \text{when } h(x) > 0 \end{cases}$$

- The state space $\mathbb{R}^n$ is split into two regions R_1 and R_2 separated by a hyper-surface Σ. Where,

$$R_1 = fx\ 2\ \mathbb{R}^n\ j\ h(x) < 0g,$$
$$R_2 = fx\ 2\ \mathbb{R}^n\ j\ h(x) > 0g,$$
$$\Sigma = fx\ 2\ \mathbb{R}^n\ j\ h(x) = 0g.$$



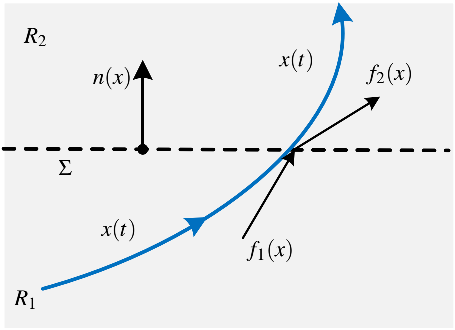
- *Filippov convex method:* the vector eld on the surface of discontinuity is a convex combination of the vector elds in the di erent regions of the state space.

Filippov First Order Theory

- Filippov first order theory defines the vector field if the solution approaches the discontinuous surface.
- The trajectory of $\dot{x} = f_1(x)$, with $x(0) = x_0$ reaches at Σ in finite time.
- Let $n(x)$ is the unit normal to Σ at x i.e. $n(x) = \frac{h_x(x)}{k h_x(x) k}$ where, $h_x(x) = r\, h(x)$ and $r = \frac{@}{@x}$.
- **Transversal Crossing:** If at $x \ 2 \ \Sigma$,

  $$(n^T(x) f_1(x)).(n^T(x) f_2(x)) > 0,$$

  leave Σ:
  - if $n^T(x) f_1(x) > 0$, to R_2,
  - if $n^T(x) f_1(x) < 0$, to R_1.



[I]

Filippov Theory: Sliding

- **Sliding:**

$$(n^T(x)f_1(x)).(n^T(x)f_2(x)) < 0$$

  - Attracting: Have existence and uniqueness (a_1 in Fig. [II]):

$$(n^T(x)f_1(x)) > 0 \text{ and } (n^T(x)f_2(x)) < 0$$

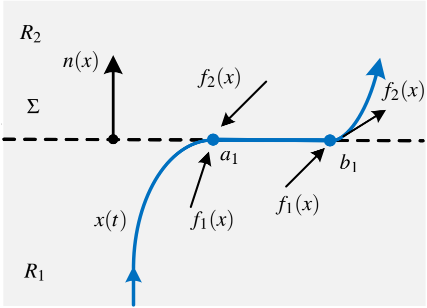
  - Repulsive: No uniqueness. Not covered.

- **Filippov vector field:** While sliding along Σ, time derivative f_F is given by,

$$f_F(x) = (1 \quad \alpha(x))f_1(x) + \alpha(x)f_2(x)$$

$$\alpha(x) = \frac{n^T(x)f_1(x)}{n^T(x)(f_1(x) - f_2(x))}$$

- Exit: during sliding, if one of the vector fields starts to point away, the solution continues above or below the sliding surface.



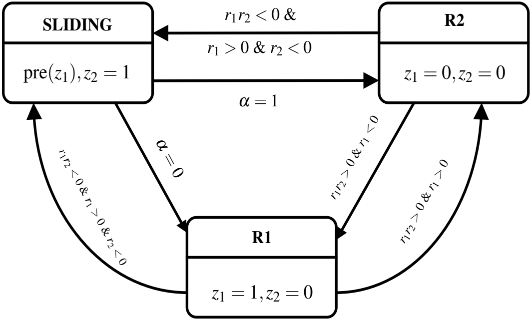
[II]

Proposed Modeling Formulation

- According to FT, a system can have three states: $h(x) < 0$ (R1). $h(x) > 0$ (R2), and $h(x) = 0$ (SLIDING).
- To implement in Modelica, we introduce two discrete variables: say z_1 and z_2, into the differential equations, as follows:

$$\dot{x} = f_1(x)\, z_1(1 \quad z_2) + f_2(x)(1 \quad z_1)(1 \quad z_2) + f_F(x) z_2$$

- Depending on the values of z_1 and z_2 (e.g. 1 or 0), a proper vector field is activated.
- In the SLIDING state the value of $z_2 = 1$.
- SLIDING deactivates both $f_1(x)$ and $f_2(x)$, without the need of changing the value of z_1. So the previous value ($\mathrm{pre}(z_1)$) is retained.



- Consider the two-dimensional Stick-slip system:

$$\dot{x} = f(x) = \begin{cases} f_1(x) \text{ when } h(x) < 0 \\ f_2(x) \text{ when } h(x) > 0 \end{cases}$$

where $h(x) = x_2 \quad 0.2$ with

$$f_1(x) = \begin{pmatrix} x_2 \\ x_1 + \frac{1}{1{:}2 \quad x_2} \end{pmatrix},$$

$$f_2(x) = \begin{pmatrix} x_2 \\ x_1 \quad \frac{1}{0{:}8 + x_2} \end{pmatrix},$$

- Simulation issues are observed with a direct implementation of this system using Modelica in OpenModelica and Dymola.

- **OpenModelica:** *Chattering detected around time 0.221654558425..0.221654756475 (100 state events in a row with a total time delta less than the step size 0.001).*
- **Dymola:** DASSL fails to continue the simulation. However RkFix2 and Euler allows to continue exposing chattering.
- Due to chattering during the simulation, the results are not mathematically accurate.
- It is not possible to understand the dynamic behavior of the real physical system.

Case Study I: Stick-slip System

- Using this FT based implementation the simulation of this system can be successfully carried out in both OpenModelica and Dymola without numerical issues.

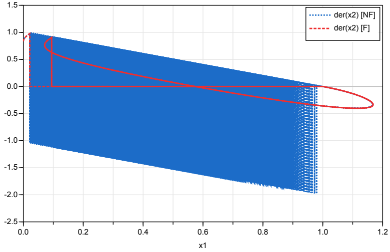


Figure 1: Time derivative of state variable ($\underline{x}_1$) of stick-slip system without (NF) and with (F) Filippov theory simulated in Dymola.

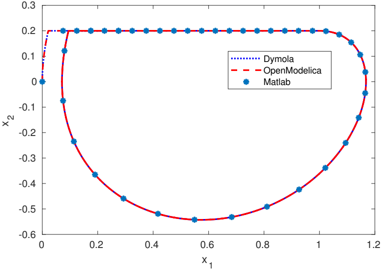


Figure 2: Periodic trajectories of the stick-slip system obtained in di erent simulation software tools.

Case Study II: A Realy Feedback System

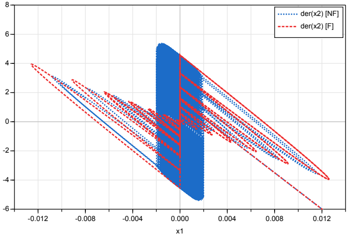
- A relay feedback system with single-input and single-output is as follows:

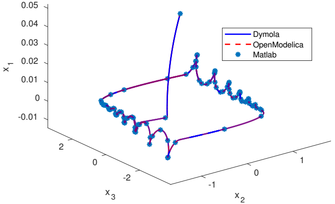
$$\dot{x} = f(x) = \begin{cases} f_1(x) & \text{when } h(x) < 0 \\ f_2(x) & \text{when } h(x) > 0 \end{cases}$$

where $h(x) = x_1$ with

$$f_1(x) = \begin{pmatrix} (2\zeta\omega + 1)x_1 + x_2 + 1 \\ (2\zeta\omega + \omega^2)x_1 + x_3 \quad 2\sigma \\ \omega^2 x_1 + 1 \end{pmatrix},$$

$$f_2(x) = \begin{pmatrix} (2\zeta\omega + 1)x_1 + x_2 \quad 1 \\ (2\zeta\omega + \omega^2)x_1 + x_3 + 2\sigma \\ \omega^2 x_1 \quad 1 \end{pmatrix}.$$





Case Study III: Anti-windup PI in an SMIB

- DAEs:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}; \mathbf{y})$$
$$\mathbf{0} = \mathbf{g}(\mathbf{x}; \mathbf{y})$$

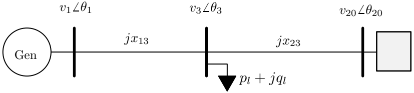
- $h(\mathbf{x}) = k_p v_a + x_i \quad v^{\max}$

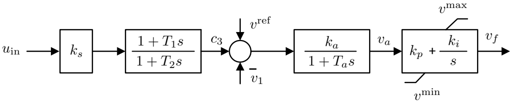
- $\mathbf{f_1}(\mathbf{x}; \mathbf{y})$:

$$\dot{\delta} = \omega$$
$$\dot{\omega} = \frac{1}{M}(p_m \quad p_e \quad D\omega)$$
$$\dot{e}_q^{0} = \frac{1}{T_{d0}^{0}}(v_f \quad \frac{x_d}{x_d^{0}} e_q^{0} + \frac{x_d \quad x_d^{0}}{x_d^{0}} v_1 \cos(\delta \quad \theta_1))$$
$$\dot{v}_a = (k_a(v^{\text{ref}} + c_3 \quad v_1) \quad v_a)/T_a$$
$$\dot{x}_i = k_i v_a$$
$$\dot{s}_1 = \frac{1}{T_2}(c_2 \quad s_1)$$

- $\mathbf{f_2}(\mathbf{x}; \mathbf{y})$:

$$\dot{e}_q^{0} = \frac{1}{T_{d0}^{0}}(v^{\max} \quad \frac{x_d}{x_d^{0}} e_q^{0} + \frac{x_d \quad x_d^{0}}{x_d^{0}} v_1 \cos(\delta \quad \theta_1))$$
$$\dot{x}_i = 0$$





Case Study III: Implementation Using Filippov Theory

- Calculating, $h_x(x) = [\frac{@h(x)}{@x_1} \quad \frac{@h(x)}{@x_2} \ ... \ \frac{@h(x)}{@x_6}]^T = [0\ 0\ 0\ k_p\ 1\ 0]^T$.
- On the switching manifold,

$$h_x^T(x) f_1(x,y) = k_p((k_a(v^{\text{ref}} + c_3 \quad v_1) \quad v_a)/T_a \ ) + k_i v_a \ ,$$
$$h_x^T(x) f_2(x,y) = k_p((k_a(v^{\text{ref}} + c_3 \quad v_1) \quad v_a)/T_a \ ) \ .$$

- If an attractive sliding occurs on Σ, then $\alpha(x,y)$ is given by:

$$\alpha(x,y) = \frac{k_p((k_a(v^{\text{ref}} + c_3 \quad v_1) \quad v_a)/T_a \ ) + k_i v_a}{k_i v_a} \ .$$

- Thus during the sliding:

$$f_F(x,y) = \quad k_p((k_a(v^{\text{ref}} + c_3 \quad v_1) \quad v_a)/T_a \ ) \ .$$

- These expressions are used in the Modelica implementation.

Case Study III: Results of SMIB System

- The SMIB system is implemented considering the FT-based formulation and a deadband based method in Modelica.
- Disturbance: step change in the voltage reference set-point ($v^{\mathsf{ref}} = 1.01$) and load ($p_{l0} = 0.71$ pu, $q_{l0} = 0.016$ pu) at $t = 5$ s.

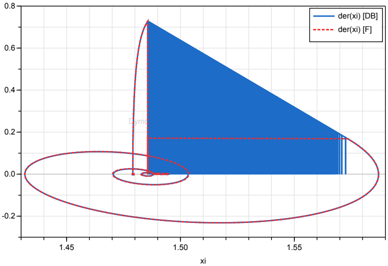


Figure 3: Time derivative of the integrator state variable ($\underline{x}_i$) in the AW PI controller with respect to the state variable (x_i) using DB and Filippov (F) methods simulated in Dymola.

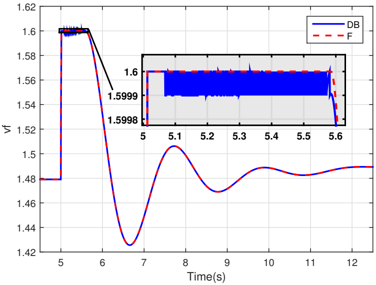


Figure 4: Trajectories of the eld voltage (v_f) using DB and Filippov (F) methods simulated in Dymola.

Conclusions and Future Work

- A generic formulation to implement Filippov system models with sliding motion using Modelica is proposed.
- Three examples are presented considering such a general-purpose design and implementation details are given.
- Simulation results in different Modelica tools indicate accurate dynamic response without any chattering or simulation halt.
- **Future Work:**
  - Future work will extend the FT-based design for multiple discontinuity surface.
  - Study the advantages from computational point of view.

The case studies are posted on-line!

`https://github.com/ALSETLab/Modelica_Fillipov_Sliding_Models`

Thank you!